Modeling and Estimation of Energy-Based Hyperelastic Objects

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Abstract

- present a method to model hyperelasticity that is well suited for representing the nonlinearity of real-world objects
- rely on a general energy-based model
- combine energy-based model with an optimization method to estimate model parameters from force-deformation examples
- FEM Simulation

A General Model of Hyperelasticity

Basis

- 形变梯度： $\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}$

- 应变能密度函数： $\Psi(\mathbf{X}) = f(\mathbf{F}(\mathbf{X}))$

- 格林应变张量：

Green strain tensor $\mathbf{E} = \frac{1}{2}\left(\mathbf{F}^T\mathbf{F} - \mathbf{I}\right)$, i.e., $\Psi(\mathbf{X}) = f(\mathbf{E}(\mathbf{X}))$

Additive Energy Model

- Additive Energy Model：

$$\Psi(\mathbf{X}) = \sum_{k} \Psi_k(\mathbf{e}_k(\mathbf{X})), \tag{1}$$

- In practice, we use unidimensional or bidimensional e_k

Additive Energy Model

- Example
  - StVK Model

$$\Psi = \frac{\lambda}{2}\,\mathrm{tr}(\mathbf{E})^2 + \mu\,\mathrm{tr}(\mathbf{E}^2)$$

$$\Psi = \underbrace{\left(\frac{\lambda}{2}+\mu\right)E_{11}{}^2}_{\Psi_1} + \underbrace{\left(\frac{\lambda}{2}+\mu\right)E_{22}{}^2}_{\Psi_2} + \underbrace{\lambda E_{11}E_{22}}_{\Psi_3} + \underbrace{2\mu E_{12}{}^2}_{\Psi_4}. \quad (3)$$

Interpolated Energy Functions

$$\Psi_k(\mathbf{X}) = \sum_s \phi(\mathbf{e}_k(\mathbf{X}) - \mathbf{e}_k^{(s)})\Psi_k^{(s)}, \tag{2}$$

where ϕ denotes some basis function, $\mathbf{e}_k^{(s)}$ a particular sample of the strain component $\mathbf{e}_k$ and $\Psi_k^{(s)}$ its corresponding weight.

Interpolated Energy Functions

- Anisotropy, simply by using different functions for the stretch energy addends $Y1$ and $Y2$
- Volume/Area preservation
- Strain/Energy-Limiting Constraints
- Heterogeneity
- the parameter set can be progressively refined to circumvent local minima

FEM Simulation

Material Estimation

Data-Driven Material Estimation

- a general deformable object with a vector q that concatenates all its nodal positions, and a vector f that concatenates all nodal forces
- input a set of N example deformations in static equilibrium
- each example deformations is produced from some known boundary conditions (forces f_c and positions q_c)
- each example contains some known measurements $\overline{m}$
  - positions, forces, or image intensities

Data-Driven Material Estimation

- Given a set of material parameters p, consisting of energy control points, and their fixed positions in strain-space

Data-Driven Material Estimation

- Objective Function

$$f_{obj} = \frac{1}{2} \sum_{i=1}^{N} w_i \left\| \mathbf{m}_i([\mathbf{q}_i, \mathbf{f}_i]\,(\mathbf{p}, \mathbf{f}_{c,i}, \mathbf{q}_{c,i})) - \bar{\mathbf{m}}_i \right\|^2 . \qquad (6)$$

Data-Driven Material Estimation

- optimization framework iterates the following three steps until convergence:
  - Update the parameter set through local optimization
  - Project parameters to enforce energy convexity
  - Simulate all examples to static equilibrium

Data-Driven Material Estimation

- Parameter Estimation *
- Static Equilibrium *

Convexity

Energy Convexity

- An energy function is convex iff the eigenvalues of its Hessian are always positive.
- For bilinear addends, the Hessian is constant;
- while for cubically interpolated unimodal addends, it is piecewise linear. This allows us to limit the enforcement of a convex energy Hessian to the control points of the energy function.

Energy Convexity

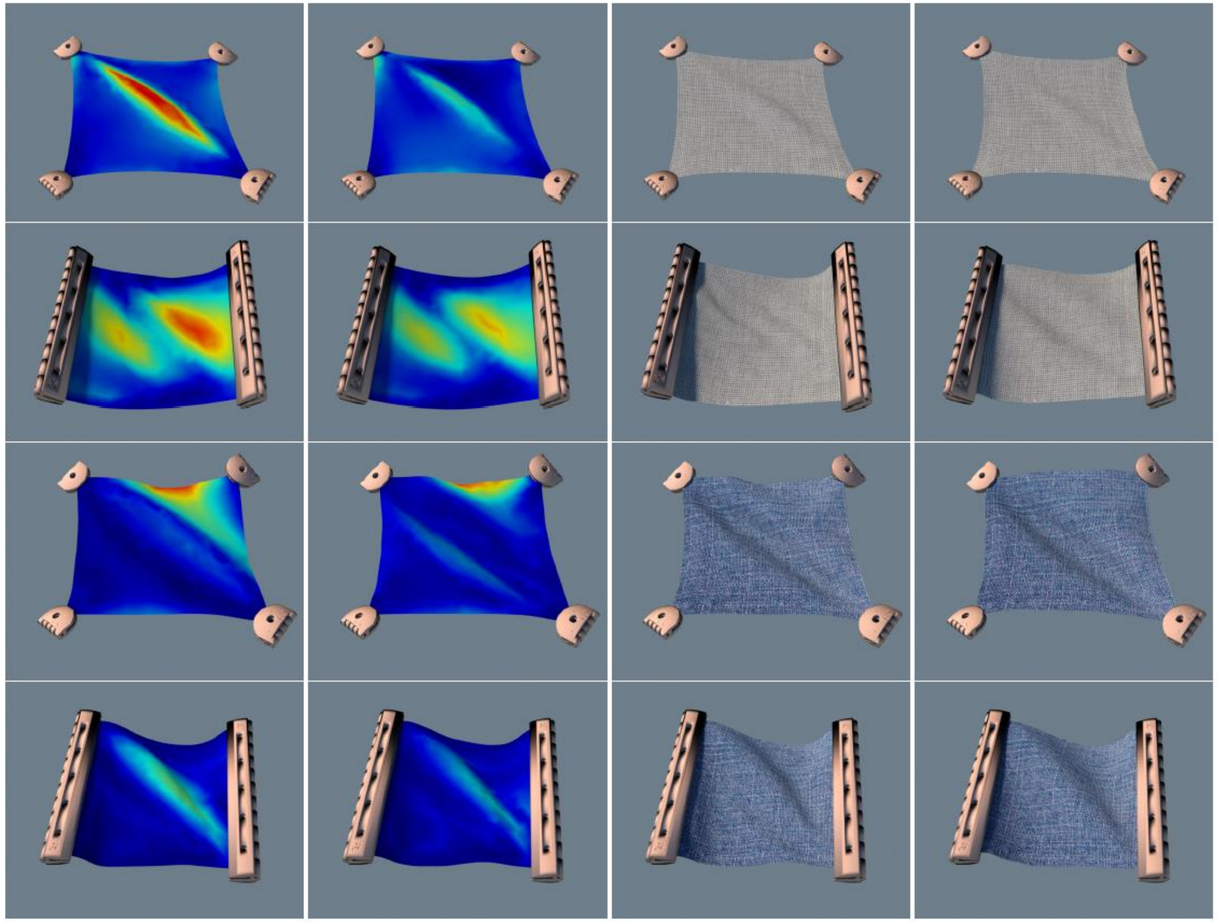
- simply enforce convexity on control points of the energy function

After an unconstrained parameter update, we test the convexity of the total energy on all control points. If some eigenvalue of the energy is negative at some control point, we project the parameter set along the gradient of the eigenvalue until it becomes positive. We implement this projection as an iterative search along the projection direction.

Results

Cloth Models from Force-Deformation Data

- Evaluation of the fitting quality of our energy model on knit (top half) and denim (bottom half) cloth deformations from [MBT*12].
- The first and third rows correspond to corner-pull motions, and the second and fourth to complex shear.
- The first column shows fitting error with a regular StVK model;
- the second column the error with our anisotropic, asymmetric, nonlinear model;
- the third column the rendered result with our model;
- and the fourth column the input deformation for reference.



- Thanks